

Total No. of Questions : 11]

SEAT No. :

PC-5147

[Total No. of Pages : 5

[6351]-111

F.E.

BSC - 101-BES : ENGINEERING MATHEMATICS - I
(2024 Pattern) (Semester - I)

Time : 2½ Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) *Q. 1 is Compulsory.*
- 2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9, Q.10 or Q.11.*
- 3) *Neat diagrams must be drawn wherever necessary.*
- 4) *Figures to the right indicate full marks.*
- 5) *Use of electronic pocket calculator is allowed.*
- 6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions. **[10]**

i) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$ is equal to _____ **[2]**

- | | |
|-------|------|
| a) 0 | b) 1 |
| c) -1 | d) 2 |

ii) if $u = x^y$ then $\frac{\partial u}{\partial x}$ is equal to _____ **[2]**

- | | |
|-----------------|------------------|
| a) 0 | b) yx^{y-1} |
| c) $x^y \log x$ | d) None of these |

iii) If $x = r \cos \theta$, $y = r \sin \theta$ then the value of $\frac{\partial(x, y)}{\partial(r, \theta)}$ is _____ **[2]**

- | | |
|------|------------------|
| a) r | b) $\cos \theta$ |
| c) 1 | d) $\frac{1}{r}$ |

P.T.O.

iv) Rank of matrix $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is _____ [2]

- a) 4 b) 3
c) 2 d) 1

v) if $A^{-1} = A^T$ then, matrix A is [1]

- a) Orthogonal b) Singular
c) Regular d) None of above

vi) The eigen values of matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & -2 & 2 \end{bmatrix}$ are _____ [1]

- a) $-2, 3, -1$ b) $-1, -2, -3$
c) $1, 2, 3$ d) $2, -2, 1$

Q2) a) Prove that, is $0 < a < b$ $\frac{b-a}{1+b^2} < \tan^{-1}(b) - \tan^{-1}(a) < \frac{b-a}{1+a^2}$. [4]

b) Expand $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. [4]

c) Find the Fourier series for the function, $f(x) = x^2$, $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$. [4]

OR

Q3) a) Verify Rolle's theorem for the function $f(x) = x^2, x \in (-1, 1)$. [4]

b) Expand $\sqrt{1 + \sin x}$ upto x^6 . [4]

c) Obtain the half range sine series for the function $f(x) = 2-x$, $0 < x < 2$. [4]

Q4) a) if $u = \log(x^2 + y^2)$, verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ [4]

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} (\tan^3 u - \tan u). \quad [4]$$

- c) If $z = f(u, v)$ and $u = \log(x^2 + y^2)$, $v = y/x$ then show that
- $$x \frac{\partial z}{\partial y} - y \frac{\partial z}{\partial x} = (1 + v^2) \frac{\partial z}{\partial v}.$$
- [4]

OR

- Q5) a)** If $u = 2x + 3y$, $v = 3x - 2y$ then find the value of $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial x}{\partial u}\right)_y \left(\frac{\partial v}{\partial y}\right)_u$
- [4]

- b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4 \sin^2) \sin^2 u.$$
 [4]

- c) If $u = f(x - y, y - z, z - x)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
- [4]

- Q6) a)** Examine the functional dependence for $u = \sin^{-1}x + \sin^{-1}y$; [4]

$v = x\sqrt{1 - y^2} + y\sqrt{1 - x^2}$, if dependent, find relation between them.

- b) Examine maxima and minima for $f(x, y) = 4x - x^2 - y^2$, also find the extreme value of the function. [4]

- c) If $u^3 + v^3 + x + y = 0$ and $x^3 + y^3 + u + v = 0$ then find $\frac{\partial u}{\partial y}$. [4]

OR

- Q7) a)** If $x + y + u^2 - v^2 = 0$ and $u + v + x^2 - y^2 = 0$ find $\frac{\partial(u, v)}{\partial(x, y)}$ [4]

- b) While calculating the volume of right circular cone, the errors of 2% and 1% are made in measuring the height and radius of the base of right circular cone respectively find the percentage error in calculated volume. [4]

- c) Find the minimum distance from origin to the plane $3x + 2y + z = 12$, using Lagrange's method of undetermined multiplier method. [4]

Q8) a) Examine consistency for the system of equations, $x + y - 3z = 1$; $4x - 2y + 6z = 8$; $15x - 3y + 8z = 20$. [4]

b) Examine for linear dependence and find relation if following vectors are dependent $(1, 1, 3)$, $(1, 2, 4)$ & $(1, 0, 2)$. [4]

c) Show that $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal matrix and hence write A^{-1} .

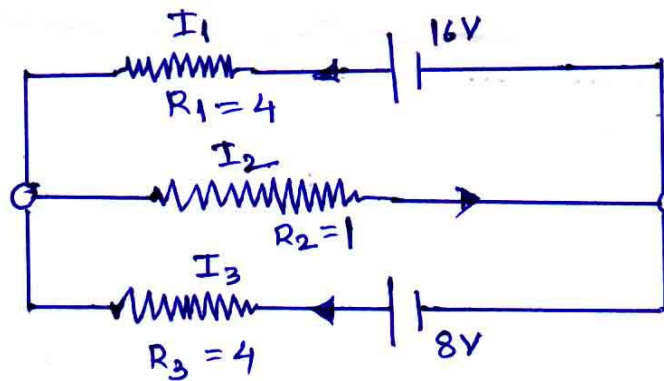
[4]

OR

Q9) a) Examine consistency for the following set of equations and obtain solution if constant $2x - y - z = 2$; $x + 2y + z = 2$; $4x - 7y - 5z = 2$. [4]

b) Examine the vectors for linear dependence and if dependent find relation between them, $(1, 1, 1)$, $(1, 2, 3)$, $(2, 3, 8)$ and $(3, 4, 9)$. [4]

c) Find the currents I_1, I_2, I_3 in the following circuit as shown in the figure. [4]



Q10) a) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Hence find A^{-1} , if it exists. [6]

b) Find modal matrix p to reduce $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ to diagonal form write the corresponding linear transformation. [6]

OR

Q11)a) Find eigen values and corresponding eigen vectors for

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad [6]$$

b) Express $Q(x) = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_3$ to canonical form by orthogonal transformation write the orthogonal transformation. [6]

