Total No. of Questions : 11]

SEAT No. : [ ]

PD4034

[Total No. of Pages : 4

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F.E.

BSC-101-BES : ENGINEERING MATHEMATICS - I

(2024 Credit Pattern) (Semester - I)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q.1 is compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9 Q.10 or Q.11.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Assume suitable data, if necessary.*
5) *Use of electronic pocket calculator is allowed.*

Q1) Write correct option for the following multiple choice questions.

a) Stationary point for the function $x^2 + y^2 + 6x + 12$ to be minimized is ________ **[2]**

i) (1, 0) ii) (3, 0)

iii) (–3, 0) iv) (0,0)

b) In the expansion of log $(1 + e^x)$ in ascending powers of x, the coefficient of 'x' is _______ **[2]**

i) 1 ii) ½

iii) ¼ iv) 0

c) If $u = \dfrac{x^2 + y^2}{x + y}$ then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is _______ **[2]**

i) u ii) 2u

iii) 0 iv) 3u

d) If $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 2 & 1 \\ 4 & -7 & -5 \end{bmatrix}$ then the rank of A is _____ **[2]**

i) 3 ii) 2

iii) 1 iv) 0

e) If A is an $n \times n$ matrix with eigen values $\lambda_1, \lambda_2 ......, \lambda_n$ the determinent of A is given by **[1]**

i) $\lambda_1 + \lambda_2 + ... + \lambda_n$

ii) the largest eigen value of A.

iii) the smallest eigen value of A.

iv) $\lambda_1, \lambda_2 ...... \lambda_n$

f) If A is 3×3 matrix with eigen values 2, 3 and 4, which of the following matrices is a diagonal matrix D in the diagonalization of A? **[1]**

i) $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

ii) $\begin{bmatrix} 0 & 2 & 3 \\ 4 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

iii) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

iv) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 3 & 4 \end{bmatrix}$

Q2) a) Verify Lagrange mean value theorem for the function $f(x) = \log x$ in the interval [1,e]. **[4]**

b) Find the fourier series for the function $f(x) = x$ in the interval $[-\pi, \pi]$.

c) Using Taylor's series expand $x^3 + 7x^2 + x - 6$ in ascending powers of $(x - 3)$. **[4]**

OR

Q3) a) Expand the function $\sqrt{1+\sin x}$ up to the term containing x^4. **[4]**

b) Using Lagrange mean value theorem, prove that

$$\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}.$$ **[4]**

c) If $f(x) = x^2$, $0 < x < 2$ then find the half range cosine series. **[4]**

OR

Q4) a) If $u = \tan^{-1}\left(\frac{x}{y}\right)$ verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **[4]**

b) If $u = \sin^{-1}\sqrt{x^2 + y^2}$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \tan^3 u$. **[4]**

c) If $z = f(x,y)$, where $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$ **[4]**

OR

Q5) a) If $x = r\cos\theta$, $y = r\sin\theta$ **[4]**

Show that $\left[x\left(\frac{\partial x}{\partial r}\right)_\theta + y\left(\frac{\partial y}{\partial r}\right)_\theta\right]^2 = x^2 + y^2$

b) If $u = \operatorname{cosec}^{-1}\sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}$, then show that **[4]**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right).$$

c) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$, prove that $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$. **[4]**

Q6) a) Examine the functional dependence, if so find the relation between them

$u = \frac{x+y}{1-xy}$; $v = \tan^{-1}x + \tan^{-1}y$ **[4]**

b) If $x = \cos\theta - r\sin\theta$, $y = \sin\theta + r\cos\theta$ then find $\frac{\partial r}{\partial x}$. **[4]**

c) Discuss the maxima and minima of the function $f(x,y) = x^2 + 4y^2 - 4x - 8y + 20$. Also find the extreme value. **[4]**

Q7) a) Given $u = x^2 - y^2$, $v = 2xy$ where $x = r\cos\theta$ $y = r\sin\theta$. **[4]**

find $\frac{\partial(u,v)}{\partial(r,\theta)}$.

b) If the kinetic energy of a body is given by $T = \frac{mv^2}{2}$ and m changes from 49 to 49.5, v changes from 1600 to 1590 then find the change in kinetic energy. **[4]**

c) Find the points on the surface $z^2 = xy + 1$ nearest to origin, by using Lagrange's method. **[4]**

Q8) a) Determine values of k, for which following system have non-trival solution $5x + 2y - 3z = 0$, $3x + y + z = 0$, $2x + y + kz = 0$. **[4]**

b) Examine for linear dependancy of following set of vectors. If dependent, find relation between them $X_1 = (3, 1, 1)$, $X_2 = (2, 0, -1)$, $X_3 = (1, 1, 2)$. **[4]**

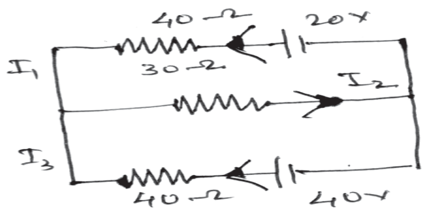
c) Show that $A = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{bmatrix}$ is orthogonal matrix and hence write A^{-1}. **[4]**

OR

Q9) a) Examine for consistency and if consistent then solve $x + y + z = 1$, $x + 2y + 4z = 2$, $x + 4y + 10z = 4$. **[4]**

b) Examine for linear dependancy of following set of vectors. If dependent find relation between them, $X_1 = (3, 1, -4)$, $X_2 = (2, 2, -3)$ $X_3 = (0, -4, 1)$. **[4]**

c) Determine the currents in the network given in the figure. **[4]**



Q10) a) Find eigen values and eigen vectors of matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ **[6]**

b) Express the quadratic form $Q(x) = 2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6zx$ to canonical form by congruent transformation. Write the corresponding linear transformation. **[6]**

OR

Q11) a) Find modal matrix p to reduce $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ to its diagonal form. **[6]**

b) By using Cayley-Hamilton theorem, find the inverse of the matrix

$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$, if it exists

Also find A^4. **[6]**

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