Total No. of Questions : 6]

SEAT No. :

PE-12172

[Total No. of Pages : 4

[6581]-2403

F.E.

BSC-101-BES : ENGINEERING MATHEMATICS - I

(2024 Pattern) (Semester - I)

Time : 2½ Hours] *[Max. Marks : 60*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Assume suitable data, if necessary.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*

Q1) Write the correct option for the following :

i) $\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos x}$ is equal to **[2]**

a) -1 b) π

c) 0 d) 3

ii) If $u = \tan^{-1}\left(\frac{x}{y}\right)$ then $u_x =$ **[2]**

a) $\frac{y}{x^2+y^2}$ b) $\frac{-x}{x^2+y^2}$

c) $\frac{1}{x^2+y^2}$ d) $\frac{xy}{1+x^2}$

iii) If $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} =$ **[2]**

a) θ b) r

c) $\frac{1}{r}$ d) 1

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iv) The rank of a matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 2 & 2\\ 3 & 3 & 3\end{bmatrix}$ is **[2]**

a) 1 b) 2

c) 3 d) 4

v) The characteristic equation of matrix $A=\begin{bmatrix}14 & -10\\ 5 & -1\end{bmatrix}$ is **[2]**

a) $\lambda^2 - 15\lambda + 25 = 0$ b) $\lambda^2 + 13\lambda - 36 = 0$

c) $\lambda^2 - 5\lambda + 10 = 0$ d) $\lambda^2 - 13\lambda + 36 = 0$

Q2) Attempt any two :

a) Using Lagrange's mean value theorem, prove that $\frac{b-a}{b} < \log\left(\frac{b}{a}\right) < \frac{b-a}{a}$

for $0 < a < b$. Hence show that $\frac{1}{4} < \ln\left(\frac{4}{3}\right) < \frac{1}{3}$. **[5]**

b) Use Taylor's theorem to expand x^3+7x^2+x-6 in powers of $(x-3)$ **[5]**

c) Prove that $e^{x\cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} - \frac{11x^4}{24} + \ldots$ **[5]**

d) Find the Fourier series to represent the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$. **[5]**

Q3) Solve any two :

a) If $u = 2\log(x-y) + \log(x+y)$ then prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}$. **[5]**

b) If $x = u\tan(v)$, $y = u\sec(v)$, then prove that $\left(\frac{\partial u}{\partial x}\right)_v \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x$ **[5]**

c) If $u = \cos^{-1}\left(\frac{xy}{\sqrt{x^2+y^2}}\right)$ then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\cot(u)\,(\operatorname{cosec}^2(u)-1)$$ **[5]**

d) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then find the value of **[5]**

$$\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$$

Q4) Solve any two :

a) If $u + v^2 = x$, $v + w^2 = y$, $w + u^2 = z$ then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **[5]**

b) Examine $u = \dfrac{x-y}{x+y}$, $v = \dfrac{x+y}{x}$ are functionally dependent. If so, find the relation between them. **[5]**

c) Find the percentage error in the area of an ellipse when an error of 2% and 1% is made in measuring its major & minor axis respectively. **[5]**

d) Discuss the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3axy$. Also find the extreme values. Note that a is positive constant. **[5]**

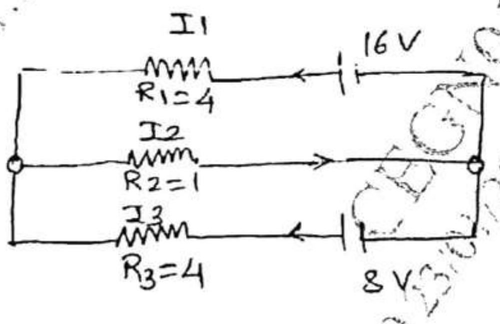
Q5) Solve any two :

a) Solve the system of equations $x + y + 2z = 0$, $x + 2y + 3z = 0$, $x + 3y + 4z = 0$. **[5]**

b) Examine the following system of vectors for linear dependence. If dependent, find relation between them. $X_1 = (1, 1, 3)$, $X_2 = (1, 2, 4)$, $X_3 = (1, 0, 2)$ **[5]**

c) Find the values of a, b, & c when $A = \dfrac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$ is orthogonal. **[5]**

d) Find the current I_1, I_2 & I_3 in the circuit as shown in the figure. **[5]**



Q6) Attempt any two :

a) Find Eigen values for the matrix A and Eigen vector corresponding to largest eigenvalue $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **[5]**

b) Verify Cayley - Hamilton theorem for matrix A and hence find A^{-1}

$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ **[5]**

c) Find the modal matrix P which diagonalises $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$ **[5]**

d) Reduce the following quadratic form to the "sum of the squares form". Find the corresponding Linear transformation

$Q(x) = 3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2xz$ **[5]**

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