Total No. of Questions : 11]

SEAT No. :

PE-12172

[Total No. of Pages : 4

[6581]-2403

F.E.

BSC-101-BES : ENGINEERING MATHEMATICS - I

(2024 Pattern) (Semester - I)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Q1 is compulsory.*
2) *Attempt Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9, Q10 or Q11.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculators is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions.

i) Let $f(x) = 4-x^2$ defined on the interval (0, 2). The value of a_0 in the fourier series of $f(x)$ is equal to **[2]**

a) $\frac{16}{3}$ b) 8

c) $\frac{8}{3}$ d) 16

ii) If $u = \log(x^2+y^2)$ then $\frac{\partial^2 u}{\partial y \partial x}$ equals to **[2]**

a) $\frac{2x}{x^2+y^2}$ b) $\frac{2y}{x^2+y^2}$

c) $\frac{4xy}{(x^2+y^2)^2}$ d) $\frac{-4xy}{(x^2+y^2)^2}$

iii) If $u = x(1-y)$, $v = xy$ then $\frac{\partial(u,v)}{\partial(x,y)} =$ **[2]**

a) y b) x

c) $\frac{1}{y}$ d) $\frac{1}{x}$

P.T.O.

iv) Rank of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 6 & 6 & 6 \end{bmatrix}$ is [2]

a) 0
b) 1
c) 2
d) 3

v) If A is an orthogonal matrix then [1]

a) $A^{-1} = 0$
b) $A^{-1} = I$
c) $A^{-1} = A'$
d) $A^{-1} = A^2$

vi) If $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 7 & 0 \\ 2 & 6 & 1 \end{bmatrix}$ then eigen values of matrix A^{-1} are [1]

a) 3, 7, 1
b) $\frac{1}{3}, \frac{1}{7}, 1$
c) 9, 49, 1
d) 0, 0, 0

Q2) a) If a<1, b<1 and a<b then prove that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}$ [4]

b) Expand $x^3 + 7x^2 + x - 6$ in powers of $(x-3)$. [4]

c) Find the Fourier series for the function $f(x) = x - x^3$ in the interval $-\pi \le x \le \pi$. [4]

OR

Q3) a) Verify the Lagrange's mean value theorem for the function $f(x) = x^2+3x+2$ in [1, 2]. [4]

b) Expand $(1+x)^{1/x}$ upto the term containing x^2. [4]

c) Obtain the half range cosine series for the function $f(x) = x^2$, $0<x<1$. [4]

Q4) a) If $u = \log(x^2+y^2) - \log x - \log y$, then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ [4]

b) If $x = e^u \tan v$, $y = e^u \sec v$, then prove that

$$\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) \cdot \left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right) = 0$$ [4]

c) If $u = x^2 - y^2$ and $v = 2xy$ and $z = f(u, v)$. Then prove that

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2\sqrt{u^2+v^2}\,\frac{\partial z}{\partial u}$$ [4]

OR

Q5) a) If $x = r\cos\theta$ and $y = r\sin\theta$ then prove that

$$\frac{1}{r}\left(\frac{\partial x}{\partial \theta}\right)_r + \left(\frac{\partial r}{\partial x}\right)_y = r\left(\frac{\partial \theta}{\partial x}\right)_y + \left(\frac{\partial x}{\partial r}\right)_\theta$$

[4]

b) If $u = \dfrac{x^3 + y^3}{y\sqrt{x}} + \dfrac{1}{x^7}\sin^{-1}\left(\dfrac{x^2 + y^2}{x^2 + 2xy}\right)$ then find the value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

[4]

c) If $u = f(x-y,\ y-z,\ z-x)$ then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$ [4]

Q6) a) If $x = uv$, $y = \dfrac{u+v}{u-v}$, then find the value of $\dfrac{\partial(u,v)}{\partial(x,y)}$. [4]

b) If $u = \sin^{-1}(x) + \sin^{-1}(y)$ and

$v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$, then determine whether u,v are functionally dependent? If yes, find the relation. [4]

c) If $x = u + e^{-v}\sin u$

$y = v + e^{-v}\cos u$

then find $\dfrac{\partial u}{\partial y}$

[4]

OR

Q7) a) If $u^2 + xv^2 - xy = 0$ and $u^2 + xyv + v^2 = 0$ then find $\dfrac{\partial u}{\partial x}$. [4]

b) Find the stationary points of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ to be maximum or minimum. [4]

c) While calculating the volume of right circular cylinder, the error of 2% and 3% are made in measuring the height and radius of the base of right circular cylinder respectively. Find the percentage error in calculated volume.

[4]

Q8) a) Check whether the following system of equations is consistent or not?
$x+y-3z = -1$; $4x - 2y + 6z = 8$; $15x - 3y + 9z = 21$. [4]

b) Examine for linear dependence. If dependent, find relation amongst the following vectors $X_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $X_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, $X_3 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ [4]

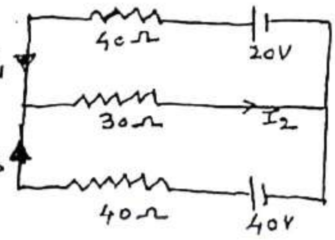
c) Show that the matrix A is orthogonal $A=\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$. **[4]**

OR

Q9) a) Solve the following system of equations by Matrix method.
$x + y + z = 3;\ x + 2y + 3z = 4,\ x + 4y + 9z = 6$ **[4]**

b) Examine the Linear dependence for the vectors (3, 1, 1), (2, 0, –1) and (4, 2, 1) **[4]**

c) Find the current in the following network. **[4]**



Q10) a) Verify Cayley - Hamilton theorem for $A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. Hence find A^{-1} if it exist. **[6]**

b) Find the modal matrix P which diagonalizes matrix $A=\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ also write corresponding linear transformation. **[6]**

OR

Q11) a) Find eigenvalues and corresponding eigenvectors for $A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$. **[6]**

b) Reduce the following quadratic form to the "Sum of squares form" find the corresponding linear transformation $2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6zx$. **[6]**

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